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Second-Order Statistics for Wave Propagation Through Complex Optical Systems

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In this report, closed form expressions are derived for various statistical functions that arise in optical propagation through arbitrary optical systems that can be characterized by a complex ABCD matrix in distributed random inhomogeneities along the optical path. Specifically, within the second-order Rytov approximation, explicit general expressions are presented for the mutual coherence function, the log-amplitude and phase correlation function, and the mean square irradiance that is obtained in propagation through an arbitrary paraxial ABCD optical system containing Gaussian-shaped limiting apertures. Additionally, we consider the performance of adaptive-optics systems through arbitrary real paraxial ABCD optical systems and derive an expression for the mean irradiance of an adaptive-optics laser transmitter through such systems.

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CONTENTS

FIGURES

1. INTRODUCTION

Recently, a useful generalized form of paraxial wave optics has been developed that can treat paraxial wave propagation through any optical system that can be characterized by a complex ABCD ray matrix.[1,2] It has been shown how Huygens' integral for propagation through a cascaded series of conventional elements, including Gaussian-shaped limiting apertures located arbitrarily along the optical path, can be accomplished in one step, using the appropriate (complex) ABCD matrix elements for that system. This approach has been useful for handling complicated multielement optical resonators, as well as resonators with the useful variable-reflectivity mirrors that are now being developed.[1]

In Ref. 2, this approach has been generalized to the case of propagation of partially coherent light and link propagation, including the effects of tilt and random jitter of the optical elements and distributed random inhomogeneities along the optical path (e.g., clear air turbulence and aerosols). However, the results presented in Ref. 2 for propagation in the presence of random inhomogeneities were limited to a real ABCD system (i.e., where all the ray matrix elements are real). Therefore, the results of Ref. 2 are limited to propagation through random media where no significant apertures or stops are located between the input and output planes of the optical system. Additionally, for propagation between conjugate image planes, the overall B matrix element is identically zero (for real ABCD systems), and some of the expressions in Section 7 and Appendix C of Ref. 2 must be applied carefully because of the apparent singularities that result. We will show that if the diffractive effects of the optical elements are included in the analysis, no such singularities occur, and all the expressions yield finite results.

Thus, the major thrust of this report is to consider propagation through a random medium in a general complex ABCD paraxial optical system. Explicit expressions are derived for the resulting second-order statistical moments of the complex optical field. In particular, we derive a general expression for the mean irradiance of a point source, the mutual coherence function, the log-amplitude, the phase and wave structure function, and the variance of irradiance for weak scintillation conditions. In Section 2, we discuss some general features of the scattered field through terms of second order in the fluctuating part of the index of refraction. In Section 3, we discuss the propagation of a point source and obtain an explicit expression for the mean irradiance of a point source propagating through a complex ABCD system. In Section 4, we present explicit results for various second-order moments of the complex field. In Section 5, we present some specific results for the mean square irradiance of a spherical wave for the Kolmogorov spectrum. We also give explicit results for the conjugate image plane and Fourier transform plane propagation geometries. Finally, in Section 6, we consider adaptive optics through real ABCD systems and derive an expression for the mean irradiance of an adaptive-optics laser transmitter through such systems.

The analysis in this report and in Ref. 2 applies primarily to optical systems that contain apertures of Gaussian-shaped variations in amplitude transmission across the optic axis. However, the analysis also applies, at least as a first approximation, to any transversely varying system whose transmission has a quadratic variation to first order near the optic axis.[1] Thus, any "soft" aperture whose transmission varies with transverse coordinate, at least near the axis, in the approximate form

$$T(x) \approx T_o(1 - a_2 x^2/2)$$

where T_o is the on-axis transmission and the coefficient a_2 is given by

$$a_2 = -\frac{1}{T_o}\left[\frac{d^2T(x)}{dx^2}\right]_{x=0}$$

can be approximated to first order by a Gaussian aperture and thus by a complex ABCD matrix. As a result, the complex paraxial analysis is a good approximation as long as the resulting beam wave remains sufficiently close to the axis so that $|a_2 x^2/2| << 1$ across the main portion of the beam.

In contrast to propagation through real ABCD systems, the statistical moments of the field that result after propagating through a complex ABCD system are not spatially homogeneous. That is, the statistical moments depend explicitly on the variables of interest rather than on the corresponding difference variables. This dependence occurs because the introduction of finite limiting apertures destroys the spatial symmetry of the field of a point source. For example, after propagating through a complex ABCD system, the mean irradiance distribution of a point source is no longer uniformly distributed over a sphere; rather, it is limited in space (i.e., it becomes a beam wave). As a result, all statistical moments of the complex field are, in general, explicit functions of absolute position variables rather than of corresponding difference variables.

In this report, we consider the case where the intervening space between optical elements is nearly that of free space with a refractive index given by $n \approx 1 + n_1$. We assume the random quantity n_1 to have zero mean and a root mean square value much less than unity. The present analysis is thus restricted to a weakly inhomogeneous medium with an electrical conductivity equal to zero and a magnetic permeability equal to one. We further assume that the characteristic scale length of the inhomogeneities is much greater than the optical wave length and that the characteristics of the medium do not change appreciably during a period of oscillation of the electromagnetic field. For example, at near IR and optical wavelengths, this condition is satisfied in the atmosphere. Additionally, we consider the propagation of scalar monochromatic fields and omit the explicit time dependence in the formulation presented below.

Finally, for simplicity in notation, we consider cylindrically symmetric optical systems, because the extension to orthogonal systems is straightforward. In cylindrically symmetric optical systems, the ray transfer matrix M is a 2×2 matrix given by

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{1.1}$$

where A, B, C, and D are complex numbers. For the case considered here, where the input and output planes are in free space

$$\text{Det}\ (M) = AD - BC = 1 \qquad (1.2)$$

2. GENERAL CONSIDERATIONS FOR THE SCATTERED FIELD

In the intervening space between the optical elements, the scalar wave equation is

$$\nabla^2 U + k^2 n^2(\vec{r})U = 0 \tag{2.1}$$

where U is a typical component of the field, k is the optical wave number, and

$$n(\vec{r}) = 1 + n_1(\vec{r}) \tag{2.2}$$

where $\langle n_1 \rangle = 0$ and $|n_1| \ll 1$. We indicate the statistical average by angular brackets and the spatial coordinate $\vec{r} = (\underline{r}, z)$, where z is assumed to be along the optical axis and $\underline{r}$ is a two-dimensional vector transverse to the optical axis. As indicated in Fig. 1, we assume, without loss of generality, that the input plane is located at $z = 0$ and the output plane is at the plane $z = L$.

We employ the Rytov transformation,[3] which consists of setting $U = \exp[\psi]$ in Eq. (2.1) and thus obtaining

$$\nabla^2\psi + (\nabla\psi)^2 + k^2[1 + n_1(\vec{r})]^2 = 0$$

It is customary to seek a solution for ψ as a power series in $\varepsilon_1 = (2n_1 + n_1^2)$

$$\psi = \sum_{m=0}^{\infty} \psi_m$$

where ψ_0 is zero order in ε_1, ψ_1 is first order in ε_1, and so forth.

In this report, we include terms through second order in ε_1. The second-order Rytov approximation

$$U(\vec{r}) = \exp[\psi_0(\vec{r}) + \psi_1(\vec{r}) + \psi_2(\vec{r})] \tag{2.3}$$

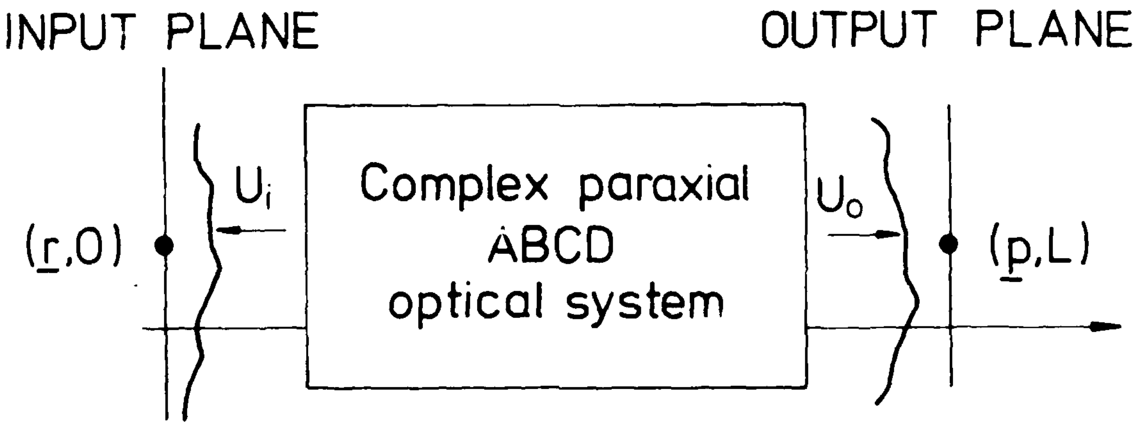

Fig. 1. Schematic Representation of Propagation through an Arbitrary ABCD Optical System

which contains all terms through second order in ϵ_1, is the lowest order non-trivial approximation that conserves energy to the order of the approximation and that gives the correct average field and the correct phase and intensity statistics.[4]

Now $U_0 = \exp \psi_0$ is the field in the absence of the inhomogeneities and is given in the paraxial approximation by[2]

$$U_0(\underline{p}_0, L) = -\frac{ik}{2\pi B} \exp(-ikL) \int d^2r \, U_i(\underline{r}) \exp\left[-\frac{ik}{2B}(Dp^2 - 2\underline{r}\cdot\underline{p} + Ar^2)\right] \quad (2.4)$$

where U_i is the field in the initial plane and A, B, and D are the ray matrix elements of the overall system between the input and output planes. The quantity L appearing on the right-hand side of Eq. (2.4) is the optical path length of a ray traveling along the optic axis. Because of the approximations used, we assume that this quantity is equal to the total propagation distance between the input and output planes.

To within an arbitrary constant, the general solution for ψ_1 is given by

$$\psi_1 = \frac{U_1}{U_0} \quad (2.5)$$

where U_1 is the first-order Born approximation

$$U_1(\vec{r}) = -k^2 \int d^3r_1 \, G(\vec{r},\vec{r}_1) \, \epsilon_1(\vec{r}_1) U_0(\vec{r}_1) \quad (2.6)$$

where

$$\epsilon_1(\vec{r}) = 2n_1(\vec{r}) + n_1^2(\vec{r}) \quad (2.7)$$

The integration in Eq. (2.6) extends over the region of space where $n_1(\vec{r})$ is different from zero, and $G(\vec{r},\vec{r}_1)$ is the appropriate Green function in the absence of the inhomogeneities.

For direct propagation between transverse planes located at z_1 and z_2, the Green function, in the paraxial approximation, is given by

$$G(\vec{r}_1,\vec{r}_2) = G(\vec{r}_1 - \vec{r}_2)$$

$$\approx -\frac{\exp[-ik(z_2 - z_1)]}{4\pi(z_2 - z_1)} \exp\left[-\frac{ik(\underline{r}_1 - \underline{r}_2)^2}{2(z_2 - z_1)}\right]$$

In the presence of a complex paraxial ABCD system, this Green function is now given by[1,2]

$$G(\vec{r}_1,\vec{r}_2) = -\frac{\exp[-ik(z_2 - z_1)]}{4\pi B_{z_1z_2}} \exp\left[-\frac{ik}{2B_{z_1z_2}}(A_{z_1z_2}r_1^2 - 2\underline{r}\cdot\underline{r}_2 + D_{z_1z_2}r_2^2)\right] \tag{2.8}$$

where we have employed the notation A_{z_1,z_2} to represent the A-matrix element for propagation between z_1 and z_2 and similarly for the quantities $B_{z_1z_2}$ and $D_{z_1z_2}$.

The second-order Rytov approximation is given by[4]

$$\psi_2(\vec{r}) = \frac{U_2(\vec{r})}{U_o(\vec{r})} - \frac{1}{2}\psi_1^2(\vec{r}) \tag{2.9}$$

where ψ_1 is the first-order Rytov approximation. The quantity U_2, the second-order Born approximation, is given by

$$U_2(\vec{r}) = (-k^2)^2\int d^3r_1\int d^3r_2 G(\vec{r},\vec{r}_1)\epsilon_1(\vec{r}_1)G(\vec{r}_1,\vec{r}_2)\epsilon_1(\vec{r}_2)\, U_o(\vec{r}_2) \tag{2.10}$$

where, for the complex paraxial system, U_o is given by Eq. (2.4) and G is given by Eq. (2.8).

Equation (2.3) can be used to compute all second-order statistical moments of the field, as obtained in a plane transverse to the optic axis at propagation distance L, through terms of second order in the fluctuations. We can show that all such second moments can be obtained from three basic field moments.

$$F_1 = \langle \phi_2(\underline{p}) \rangle \tag{2.11a}$$

$$F_2 = \langle \psi_1(\underline{p}_1)\psi_1^*(\underline{p}_2) \rangle \tag{2.11b}$$

and

$$F_3 = \langle \psi_1(\underline{p}_1)\psi_1(\underline{p}_2) \rangle \tag{2.11c}$$

where

$$\phi_2 = \frac{U_2}{U_o} \tag{2.12}$$

and ψ_1 is given by Eq. (2.5).

For example, the average field $\langle U \rangle = U_o \exp(\langle \phi_2 \rangle)$, the mutual coherence function, is given by

$$\Gamma(\underline{p}_1,\underline{p}_2) = \langle U(\underline{p}_1)U^*(\underline{p}_2) \rangle$$

$$\propto \exp[\langle \phi_2(\underline{p}_1) \rangle + \langle \phi_2^*(\underline{p}_2) \rangle + \langle \psi_1(\underline{p}_1)\psi_1^*(\underline{p}_2) \rangle] \tag{2.13}$$

and the normalized variance of irradiance (under weak scintillation conditions) is given by

$$\sigma_I^2(\underline{p}) = \frac{\langle I^2(\underline{p}) \rangle}{\langle I(\underline{p}) \rangle^2} - 1$$

$$\simeq \exp[4\chi^2(\underline{p})] - 1$$

where

$$\chi^2(\underline{p}) = \frac{1}{2}\,\mathrm{Re}(F_2 + F_3) \tag{2.14}$$

and Re denotes the real part.

To obtain the field in the output plane from an arbitrary field U_i in the input plane (e.g., via a Huygens-Fresnel integral), it is sufficient to calculate the quantities F_1, F_2, and F_3 for the special case where the sources of the fields are point sources. To calculate these quantities, we denote, by $\psi_1(\underline{\rho},\underline{r})$ and $\psi_2(\underline{\rho},\underline{r})$, the first- and second-order Rytov approximation to the optical field at $(\underline{\rho},L)$ due to a point source located at $(\underline{r},0)$, respectively. The calculation of the statistical quantities F_1, F_2, and F_3 is given in the Appendix, with the results that

$$F_1 = \langle \phi_2(\underline{\rho},\underline{r}) \rangle$$

$$= -\pi k^2 \int_0^L dz \int d^2K \Phi_n(\underline{K};z) \tag{2.15}$$

$$F_2 = \langle \psi_1(\underline{\rho}_1,\underline{r}_1)\psi_1^*(\underline{\rho}_2,\underline{r}_2)\rangle$$

$$= 2\pi k^2 \int_0^L dz \int d^2K \; \Phi_n(\underline{K};z)\exp(-K^2\beta_i/k)$$

$$\times \exp\{-i\underline{K}\cdot\{[\underline{\rho} \; Re(B_{0z}/B) + \underline{\rho} \; Re \; (B_{zL}/B)]$$

$$+ 2i \; [\underline{P} \; Im(B_{0z}/B) + \underline{R} \; Im(B_{zL}/B)]\}\} \tag{2.16}$$

$$F_3 = \langle \psi_1(\underline{\rho}_1,\underline{r}_1)\psi_1(\underline{\rho}_2,\underline{r}_2)\rangle$$

$$= - 2\pi k^2 \int_0^L dz \int d^2K \; \Phi_n(\underline{K};z) \; \exp(iK^2\beta/k)$$

$$\times \exp\{-i\underline{K}\cdot[\underline{\rho}(B_{0z}/B) + \underline{\rho}(B_{zL}/B)]\} \tag{2.17}$$

where $\Phi_n(\underline{K};z)$ is the three-dimensional spectrum of the index of refractive fluctuations at propagation distance z evaluated at $K_z = 0$. The quantities

B_{0z}, B_{zL}, and $B \equiv B_{0L}$ are the B-matrix elements for propagation through the system from 0 to z, z to L, and 0 to L, respectively

$$\beta = \frac{B_{0z} B_{zL}}{B} \qquad (2.18)$$

$\beta_i = \mathrm{Im}(\beta)$, where Im denotes the imaginary part, and

$$\underline{\rho} = \underline{\rho}_1 - \underline{\rho}_2 \qquad (2.19a)$$

$$\underline{P} = \frac{1}{2}(\underline{\rho}_1 + \underline{\rho}_2) \qquad (2.19b)$$

$$\underline{r} = \underline{r}_1 - \underline{r}_2 \qquad (2.19c)$$

and

$$\underline{R} = \frac{1}{2}(\underline{r}_1 + \underline{r}_2) \qquad (2.19d)$$

In general, B_{0z} and B_{zL} (and hence β) depend on z, depending on the optical system under consideration. Through these quantities, the optical system affects the second-order statistical moments of point sources. For the important case of an isotropic power spectrum (e.g., the Kolmogorov spectrum), $d^2K \to 2\pi K dK$ and

$$\exp(-i\underline{K}\cdot\underline{V}) \to J_0(K|\underline{V}|)$$

where J_0 is the zero-order Bessel function of the first kind and $\underline{V}$ is the (complex) vector within the braces in Eqs. (2.16) and (2.17). In the rest of this report, we will be dealing with spatially isotropic power spectra.

Examination of Eq. (2.15) reveals that, through terms of second order in the fluctuations, the quantity $F_1 = \langle\phi_2\rangle = \langle U_2\rangle/U_0$ is real and is independent of the observation point and the matrix elements of the optical system. In particular, Eq. (2.15) is identical to what is obtained for line-of-sight propagation over a distance L. As indicated in Eqs. (2.16) and (2.17), the quantities F_2 and F_3 depend on the specific properties of the optical system and are spatially nonhomogeneous, because they depend explicitly on the variables $\underline{r}_1$, $\underline{r}_2$, $\underline{\rho}_1$, and $\underline{\rho}_2$. However, for the special case of real ABCD systems, F_2 and F_3 are spatially inhomogeneous, because there are no limiting apertures that inhibit the propagation of spherical waves through the medium. Note that for line-of-sight propagation, $B_{0z}/B = z/L$, $B_{zL}/B = (L - z)$, and $\beta = z(L - z)/L$. In this case, Eqs. (2.16) and (2.17) become identical to the corresponding results given in the literature.[3]

Thus, Eqs. (2.15)-(2.17) contain all the effects of the arbitrary complex paraxial ABCD system on the second-order statistical properties of point sources. They are now the basis for calculating all the second-order statistical moments of interest.

3. MEAN FIELD AND IRRADIANCE OF A POINT SOURCE

Consider a point source located at the origin of coordinates, which radiates a total power W. To within a multiplicative phase factor, we can show from Eq. (2.4) that the field in the observation plane in the absence of the inhomogeneities is given by

$$U_o(p) = \left(\frac{W}{4\pi|B|^2}\right)^{1/2} \exp\left(-ikL - \frac{ikDp^2}{2B}\right) \tag{3.1}$$

from which it follows that the irradiance $I_o = |U_o|^2$ is given by

$$I_o(p) = \frac{W}{4\pi|B|^2} \exp\left[-k|\mathrm{Im}(D/B)|p^2\right] \tag{3.2}$$

For real ABCD systems, $I_o = W/4\pi B^2$, which, for line-of-sight propagation, reduces to $I_o = W/4\pi L^2$.

Examination of Eq. (3.2) reveals that in the presence of a complex paraxial ABCD optical system, the irradiance distribution of a point source has a Gaussian shape with a $1/e^2$ spot radius p_o given by

$$p_o^2 = \frac{2}{k|\mathrm{Im}(D/B)|} \tag{3.3}$$

As an elementary example, consider a point source located at distance z_1 to the left of a Gaussian lens of real focal length f and $1/e^2$ transmission radius σ (see Fig. 2). If the observation plane is in the right-hand focal plane of the lens, the results are

$$B = F\left(1 - \frac{2z_1 i}{k\sigma^2}\right) \tag{3.4}$$

and

$$D = 1 - \frac{z_1}{f} - \frac{2z_1 i}{k\sigma^2} \tag{3.5}$$

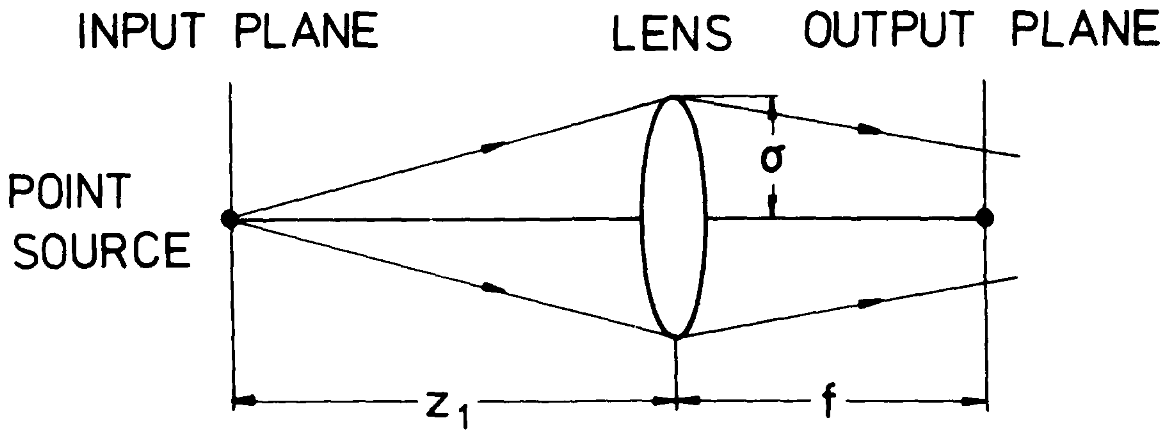

Fig. 2. Propagation Geometry of a Point Source Located a Distance z_1 to the Left of a "Thin Gaussian" Lens of Real Focal Length f and $1/e^2$ Transmission Radius σ. The output plane is assumed to be in the right-hand focal plane of the lens.

If we substitute Eqs. (3.4) and (3.5) into Eq. (3.3), the result is

$$p_o^2 = (f\sigma/z_1)^2 + (2f/k\sigma)^2 \tag{3.6}$$

The square of the $1/e^2$ beam radius is a sum of two terms: the first term, on the right-hand side of Eq. (3.6), is a geometric magnification; the second term gives the diffraction effects of the finite lens.

When inhomogeneities are present, the mean field is given by

$$\langle U(\underline{p})\rangle = U_o\langle\exp[\psi_1(\underline{p}) + \psi_2(\underline{p})]\rangle \tag{3.7}$$

If $g = \ln(y)$ is a random variable through terms of second order in the fluctuations with mean $\langle g\rangle 40$ and mean square $\langle g^2\rangle$, then correct through terms of the order of ϵ_1^2,

$$\langle y\rangle = \exp\left[\langle g\rangle + \frac{1}{2}\langle(g - \langle g\rangle)^2\rangle\right] \tag{3.8}$$

This result may be used to obtain the average of the right-hand side of Eq. (3.7). Hence, through terms of the order of ϵ_1^2 ($\approx 4n_1^2$),

$$\langle U(p)\rangle = U_o \exp(\langle\phi_2\rangle) \tag{3.9}$$

where U_o is given by Eq. (3.1) and $\langle\phi_2\rangle$ is given by Eq. (2.15). Apart from the U_o term, the average field of a point source is identical to that obtained for line-of-sight propagation through the random medium.

The mean irradiance of a point source is obtained as

$$\langle I(\underline{p})\rangle = \langle|U(\underline{p})|^2\rangle \tag{3.10}$$

If we substitute $U(p) = U_o(\underline{p}) \exp[\psi_1(\underline{p}) + \psi_2(\underline{p})]$ into Eq. (3.10) and apply Eq. (3.8) to obtain the average, the result is

$$\langle I(\underline{p})\rangle = I_o(\underline{p}) \exp[\Delta(\underline{p})] \tag{3.11}$$

where $I_0(p)$ is given by Eq. (3.2) and

$$\Delta(\underline{p}) = 2\mathrm{Re}\langle\phi_2\rangle + \langle|\psi_1(\underline{p})|^2\rangle \qquad (3.12)$$

If we substitute Eqs. (2.15) and (2.16), with $\underline{r}_1 = \underline{r}_2 = 0$ and $\underline{p}_1 = \underline{p}_2 = \underline{p}$, into Eq. (3.12), we obtain the following equation for an isotropic index of refraction power spectrum

$$\Delta(p) = 4\pi^2 k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)$$

$$[\exp(-K^2\beta_i/k) I_0(2\alpha_i pK) - 1] \qquad (3.13)$$

where

$$\alpha_i = \mathrm{Im}(B_{0z}/B) \qquad (3.14)$$

and, in Eq. (3.13), $I_0(x)$ is the zeroth-order Bessel function of the first *kind of imaginary argument.* Equations (3.10) and (3.13) are the general expressions for the mean irradiance profile of a point source propagating through an aribtrary complex ABCD system in the presence of random media, as specified by $\Phi_n(\underline{K};z)$. For real ABCD systems, $\Delta(p) = 0$, as it should because of spatial symmetry.[4] Because $I_0(x) \geq 1$, the contribution to $\Delta(p)$ from the Bessel function term is always greater than zero, indicating that the inhomogeneities broaden the irradiance pattern.

For small values of p, one can readily obtain a power series expansion of $\Delta(p)$ as

$$\Delta(p) \simeq -4\pi^2 k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)$$

$$\times\ [1 - \exp(-K^2\beta_i/k) - \alpha_i^2 p^2 K^2 \exp\,(-K^2\beta_i/k) + \ldots] \qquad (3.15)$$

Consider the Kolmogorov spectrum in the inertial subrange[3]

$$\Phi_n(K;z) = 0.033\, C_n^2(z)\, K^{-11/3} \tag{3.16}$$

where $C_n^2(z)$ is the index structure constant profile. If we substitute Eq. (3.16) into Eq. (3.15) and perform the integration over spatial frequency, the result is

$$\Delta(p) \simeq -4.25\, k^{7/6} \int_0^L dz\, C_n^2(z)[\beta_i(z)]^{5/6}$$

$$+ 3.65\, k^2 p^2 \int_0^L dz\, C_n^2(z)\, \alpha_i^2(z)\, [k/\beta_i(z)]^{1/6} + \ldots \tag{3.17}$$

For the propagation geometry illustrated in Fig. 2, with $C_n^2 \neq 0$ between the point source and the lens only, and $z_1 \gg k\,\sigma^2$, we readily obtain

$$\Delta(p) \simeq -\,(1.62\sigma/\rho_o)^{5/3}\,[1 - 1.25\,(k\sigma/2f)^2 p^2 + \ldots] \tag{3.18}$$

where

$$\rho_o = [1.46\, k^2 \int_0^{z_1} dz\, C_n^2(z)(z/z_1)^{5/3}]^{-3/5} \tag{3.19}$$

is the spherical wave lateral coherence length of a spherical wave for propagation between $z = 0$ and $z = z_1$. From Eqs. (3.11) and 3.18), we see that the first and second term on the right-hand side of Eq. (3.18) give the inhomogeneities-induced reduction in the on-axis mean irradiance and beam broadening.

4. GENERALIZED SECOND-ORDER MOMENTS

In this section, we present general expressions for various second-order statistical moments that arise in propagation through complex ABCD systems in a random medium.

4.1 CORRELATION FUNCTIONS

We have

$$\psi_1(\underline{p},\underline{r}) = \chi(\underline{p},\underline{r}) - iS(\underline{p},\underline{r}) \tag{4.1}$$

where $\chi(\underline{p},\underline{r})$ is the log-amplitude and $S(\underline{p},\underline{r})$ is the phase at a point $\underline{p}$ in the output plane resulting from a point source at $\underline{r}$ in the input plane. For simplicity in notation, we omit subscripts on both χ and S. We can then show that

$$B_\chi(\underline{p}_1,\underline{p}_2;\ \underline{r}_1,\underline{r}_2) = \langle \chi(\underline{p}_1,\underline{r}_1)\chi(\underline{p}_2,\underline{r}_2)\rangle = \frac{1}{2}\,\mathrm{Re}(F_2 + F_3) \tag{4.2}$$

$$B_S(\underline{p}_1,\underline{p}_2;\ \underline{r}_1,\underline{r}_2) = \langle S(\underline{p}_1,\underline{r}_1)S(\underline{p}_2,\underline{r}_2)\rangle = \frac{1}{2}\,\mathrm{Re}(F_2 - F_3) \tag{4.3}$$

and

$$B_{S\chi}(\underline{p}_1,\underline{p}_2;\ \underline{r}_1,\underline{r}_2) = \langle S(\underline{p}_1,\underline{r}_1)\chi(\underline{p}_2,\underline{r}_2)\rangle = \frac{1}{2}\,\mathrm{Im}(F_2F_3) \tag{4.4}$$

where B_χ is the log-amplitude, B_S is the phase, and $B_{S\chi}$ is the phase log-amplitude correlation functions.

Except for adaptive optics, discussed in Section 6, the general two-point to two-point moments are not of general interest, although they can be obtained directly by substituting Eqs. (2.16) and (2.17) into Eqs. (4.2)-(4.4). We restrict our attention to the special case of a single point source located in the input plane at $\underline{r} = 0$. Following the above procedure, we find

$$B_{\chi,S}(p_1,p_2) = 2\pi^2k^2 \int_0^L dz \int_0^\infty dK\ K\Phi_n(K;z)\exp(-K^2\beta_i/k)$$

$$\times \left\{\mathrm{Re}[J_0(K|\underline{p}\alpha_r+2i\underline{P}\alpha_i|)] \mp \cos\left(\frac{K^2\beta_r}{k}\right) J_0(Kp\alpha_r)\right\} \qquad (4.5)$$

where the $\mp$ sign refers to the log-amplitude and phase correlation functions, respectively, and

$$\alpha_r = \mathrm{Re}(B_{0z}/B) \qquad (4.6a)$$

$$\beta_r = \mathrm{Re}\beta \qquad (4.6b)$$

Similarly

$$B_{S\chi}(\underline{p}_1,\underline{p}_2) = 2\pi^2k^2 \int_0^L dz \int_0^\infty dK\ K\Phi_n(K;z)\ \exp(-K^2\beta_i/k)$$

$$\times \left\{\mathrm{Im}[J_0(K|\underline{p}\alpha_r + 2i\underline{P}\alpha_i|)] + \sin\left(\frac{K^2\beta_r}{k}\right) J_0(Kp\alpha_r)\right\} \qquad (4.7)$$

For the special case where $\underline{p}_1 = -\underline{p}_2$, Eqs. (4.5) and (4.7) become

$$B_{\chi,S}(\underline{p},-\underline{p}) = 2\pi^2k^2 \int_0^L dz \int_0^\infty dK\ K\Phi_n(K;z)\exp(-K^2\beta_i/k)$$

$$\times \left[1 \mp \cos\left(\frac{K^2\beta_r}{k}\right)\right]J_0(2Kp\alpha_r) \qquad (4.8)$$

and

$$B_{S\chi}(\underline{p},-\underline{p}) = 2\pi^2k^2 \int_0^L dz \int_0^\infty dK\ K\Phi_n(K;z)\ \exp(-K^2\beta_i/k)$$

$$\times \sin\left(\frac{K^2\beta_r}{k}\right) J_0(2Kp\alpha_r) \qquad (4.9)$$

The variance of log-amplitude, phase, and phase log-amplitude is obtained from Eqs. (4.5) and (4.6) by setting $\underline{p}_1 = \underline{p}_2$. We find

$$\left.\begin{matrix}\langle\chi^2\rangle\\ \langle S^2\rangle\end{matrix}\right\} = B_{\chi,S}(\underline{\rho},\underline{\rho})$$

$$= 2\pi^2k^2 \int_0^L dz \int_0^L dK\, K\Phi_n(K;z)\, \exp(-K^2\beta_i/k)$$

$$\times \left[I_0(2K\rho\alpha_i) \mp \cos\left(\frac{K^2\beta_r}{k}\right)\right] \tag{4.10}$$

and

$$\langle S\chi\rangle = B_{S\chi}(\underline{\rho},\underline{\rho})$$

$$= 2\pi^2k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)\, \exp(-K^2\beta_i/k)$$

$$\times \sin\left(\frac{K^2\beta_r}{k}\right) \tag{4.11}$$

4.2 MUTUAL COHERENCE FUNCTION

The mutual coherence function (MCF) is defined as

$$\Gamma(\underline{\rho}_1,\underline{\rho}_2) = \langle U(\underline{\rho}_1)U^*(\underline{\rho}_2)\rangle \tag{4.12}$$

We omit the deterministic contribution, $U_o(\underline{\rho}_1)\, U_o^*(\underline{\rho}_2)$, to the MCF. This contribution can be obtained directly from Eq. (3.1). We now have

$$U(\underline{\rho}) = \exp[\psi_1(\underline{\rho}) + \psi_2(\underline{\rho})] \tag{4.13}$$

where ψ_1 is given by Eq. (2.5) and ψ_2 is given by Eq. (2.9). If we substitute Eq. (4.13) into Eq. (4.12) and use Eq. (3.8) to obtain the statistical average of the right-hand side of Eq. (4.12), the result is

$$\Gamma(\underline{\rho}_1,\underline{\rho}_2) = \exp[2\langle\phi_2\rangle + \langle\psi_1(\underline{\rho}_1)\psi_1^*(\underline{\rho}_2)\rangle] \tag{4.14}$$

If we substitute Eqs. (2.15) and (2.16) into Eq. (4.14), we obtain

$$\Gamma(\underline{\rho}_1,\underline{\rho}_2) = \exp\{-4\pi^2 k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)$$

$$\times [1 - \exp(-K^2\beta_i/k)\, J_0(K|\underline{\rho}\alpha_r + 2\underline{P}\alpha_i|]\} \qquad (4.15)$$

In general, Γ is complex and spatially inhomogeneous with respect to $\underline{\rho}_1$ and $\underline{\rho}_2$. Equation (4.15) is the general expression for the MCF of a point source located at the origin. The equation applies for a general complex ABCD paraxial system, as specified by α and β. For real ABCD systems, Eq. (4.15) becomes identical to the corresponding results of Ref. 2.

For some applications, knowledge of the complex degree of coherence is required. This quantity is given by the normalized MCF[5]

$$\gamma(\underline{\rho}_1,\underline{\rho}_2) = \frac{\Gamma(\underline{\rho}_1,\underline{\rho}_2)}{[\Gamma(\underline{\rho}_1,\underline{\rho}_1)\Gamma(\underline{\rho}_2,\underline{\rho}_2)]^{1/2}} \qquad (4.16)$$

If we substitute Eq. (4.14) into Eq. (4.16), we obtain

$$\gamma(\underline{\rho}_1,\underline{\rho}_2) = \exp\{\langle\psi_i(\underline{\rho}_1)\psi_2^*(\underline{\rho}_2)\rangle - \frac{1}{2}[\langle|\psi_1(\underline{\rho}_1)|^2\rangle + \langle|\psi_1(\underline{\rho}_2)|^2\rangle]\} \qquad (4.17)$$

$$= \exp\{-2\pi k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)\exp(-K^2\beta_i/k)$$

$$\times [I_0(2\alpha_i p_1 K) + I_0(2\alpha_i p_2 K) - 2J_0(K|p\alpha_r + 2\underline{i}P\alpha_i|)]\} \qquad (4.18)$$

5. IRRADIANCE STATISTICS

For weak scintillation (i.e., $\chi^2 \ll 1$), it is well known that the irradiance statistics can be expressed as second-order moments of the field.[3] Here we present a general expression of the normalized variance of irradiance of a point source located at the origin

$$\sigma_I^2 = \frac{\langle I^2(\underline{p}) \rangle}{\langle I(\underline{p}) \rangle^2} - 1 \tag{5.1}$$

where

$$I(\underline{p}) = |U(\underline{p})|^2 \tag{5.2}$$

and

$$U(\underline{p}) = U_o(\underline{p}) \exp[\psi_1(\underline{p}) + \psi_2(\underline{p})] \tag{5.3}$$

If we substitute Eqs. (5.2) and (5.3) into Eq. (5.1), making use of Eqs. (4.1) and (3.8), we obtain through terms through second order in the fluctuations

$$\sigma_I^2(\underline{p}) = \exp[4\langle\chi^2(\underline{p})\rangle] - 1 \approx 4\langle\chi^2(\underline{p})\rangle \tag{5.4}$$

where $\langle\chi^2(p)\rangle$ is given by Eq. (4.10). Effects that cause beam broadening in the output plane (e.g., wave front tilt) are contained identically in $\langle I^2 \rangle$ and $\langle I \rangle^2$. Thus, the normalized variance σ_I^2 is independent of such effects and reflects the focusing/defocusing properties of the random inhomogeneities only along the optical path.

For example, consider a point source located at distance z_1 to the left of a lens of real focal length f and Gaussian radius σ. We will determine the on-axis irradiance variance at a distance z_2 to the right of the lens. We will assume that the intervening space between the lens and point source is filled with a turbulent medium, which is characterized by the Kolmogorov spectrum.

For p = 0 and the Kolmogorov spectrum [Eq. (4.10)], we obtain

$$\langle \chi^2(0)\rangle = 0.652\, k^2 \int_0^L dz\, C_n^2(z) \int_0^\infty dK\, K^{-8/3} \exp(-K^2 \beta_i/k)$$

$$\times\, [1 - \cos(K^2 \beta_r/k)] \qquad (5.5)$$

$$= 0.326\, k^{7/6} \int_0^L dz\, C_n^2(z)\, \beta_r(z)^{5/6} W(z) \qquad (5.6)$$

where

$$W(z) = \int_0^\infty \frac{dx}{x^{11/6}} (1 - \cos x) \exp[-\mu(x)x]$$

$$= \Gamma(-5/6)\{\mu^{5/6} - (1 + \mu^2)^{5/12} \cos[5/6 \arctan(1/\mu)]\} \qquad (5.7)$$

$$\mu(z) = \frac{\beta_i(z)}{\beta_r(z)} \qquad (5.8)$$

and $\Gamma(\ldots)$ is the Gamma function.[6] For real ABCD systems, $\mu = 0$ and Eq. (5.6) yields the results given in Ref. 2. For $\mu \ll 1$, Eq. (5.7) yields $W(z) \to -\Gamma(-5/6)\cos(5\pi/12) \approx 1.73$, and for $\mu \gg 1$, Eq. (5.7) yields $W(z) \to -(5/12)\Gamma(-5/6)\, \mu^{-7/6} \approx 2.78\, \mu^{-7/6}$.

Equations (5.6)-(5.8) are the general expressions for the on-axis log-amplitude variance for the Kolmogorov spectrum. These equations are valid for an arbitrary complex ABCD system, as specified by $\beta(z)$. The quantity $W(z)$ is an additional weighting function that results from the optical system. In contrast to the results for real ABCD systems,[2] no singularities result for complex ABCD systems.

To be specific, we consider two well-known examples of practical interest: imaging and Fourier transform propagation geometries.

5.1 IMAGE PLANE VARIANCE OR IRRADIANCE

Here we have

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

For simplicity, we assume that $C_n^2 \neq 0$ only between the point source and the lens. It is straightforward to show that[2]

$$B = -\frac{2iz_1z_2}{k\sigma^2} \tag{5.9a}$$

and for $z \leq z_1$

$$B_{0z} = z \tag{5.9b}$$

and

$$B_{zL} = \frac{zz_2}{z_1} - \frac{2iz_1z_2}{k\sigma^2}\left(1 - \frac{z}{z_1}\right) \tag{5.9c}$$

Thus, for $z \leq z_1$

$$\beta_r = z\left(1 - \frac{z}{z_1}\right) \tag{5.10a}$$

and

$$\beta_i = \frac{z^2 k\sigma^2}{2z_1^2} \tag{5.10b}$$

Hence, from Eq. (5.8), we obtain

$$\mu = \frac{a(z/z_1)}{1 - \frac{z}{z_1}} \tag{5.11}$$

where

$$a = \frac{k\sigma^2}{2z_1} \tag{5.12}$$

Over the range of integration in Eq. (5.6), where the integrand is non-zero, $z \sim z_1$, and, hence, $\mu \sim a$. Thus, when the point source is in the far field of the lens ($z_1 \gg k\,\sigma^2$), $\mu \gg 1$ over the range of integration. Conversely, for near-field conditions ($z_1 \ll k\sigma^2$), $\mu \gg 1$ over the range of integration. As a result, the asymptotic limits for the on-axis log-amplitude variance, for constant C_n^2, are given by

$$\langle \chi^2 \rangle = 0.564\, k^{7/6}\, C_n^2 \int_0^{z_1} dz\, \beta_r(z)^{5/6}$$

$$= 0.124\, k^{7/6}\, C_n^2\, z_1^{11/6}, \quad z_1 \gg k\sigma^2 \tag{5.13}$$

and

$$\langle \chi^2 \rangle = 0.906\, k^{7/6}\, C_n^2 \int_0^{z_1} dz\, \beta_r(z)^{5/6} [\mu(z)]^{-7/6}$$

$$= 0.612\, k^{7/6}\, C_n^2 z_1^{11/6} (2z_1/k\sigma^2)^{7/6}, \quad z_1 \ll k\sigma^2 \tag{5.14}$$

Equation (5.13) is just the spherical wave variance obtained for line-of-sight propagation over a distance z_1, whereas Eq. (5.14) is proportional to this variance multiplied by an averaging aperture factor $(z_1/k\,\sigma^2)^{7/6} \ll 1$. We can explain these results by noting that, for weak scintillation conditions, the transverse irradiance correlation length for propagation over a distance z_1 is of the order $(z_1/k)^{1/2}$. For an imaging geometry, light incident on the lens from the left converges toward the image point on the right. Then, for far-field conditions $[\sigma \ll (z_1/k)^{1/2}]$, the entire lens lies within a single correlation patch, and no aperture averaging is expected. For near-field conditions $[\sigma \gg (z_1/k)^{1/2}]$, many independent correlation patches are contained over the imaging lens, resulting in a reduced spherical wave variance at the image point. For the Kolmogorov spectrum, this reduction or aperture averaging factor is proportional to the -7/3 power of the number of independent correlation patches over the aperture.[3]

Equation (5.6) has been integrated numerically for arbitrary $a = k\sigma^2/2z_1$. In Fig. 3, we present a plot of $\langle\chi^2\rangle_a/\langle\chi^2\rangle_{a=0}$ as a function of a.

5.2 FOURIER TRANSFORM PLANE VARIANCE OF IRRADIANCE

Here, we consider the case where the point source is in the left-hand focal plane and the observation point is in the right-hand focal plane of a lens of real focal length f and Gaussian radius σ. It is straightforward to show that[2]

$$B = f\left(1 - \frac{2fi}{k\sigma^2}\right) \tag{5.15a}$$

and for $z \le f$

$$B_{0z} = z \tag{5.15b}$$

and

$$B_{zL} = B + \frac{2zfi}{k\sigma^2} \tag{5.15c}$$

Thus, for $z \le f$, we obtain

$$\beta_r = \frac{z[a^2 + (1 - \frac{z}{f})]}{1 + a^2} \tag{5.16a}$$

and

$$\beta_i = \frac{az^2}{f(1 + a^2)} \tag{5.16b}$$

where

$$a = k\sigma^2/2f$$

Hence, from Eq. (5.8), we obtain

$$\mu = \frac{a(z/f)}{a^2 + (1 - \frac{z}{f})} \tag{5.17}$$

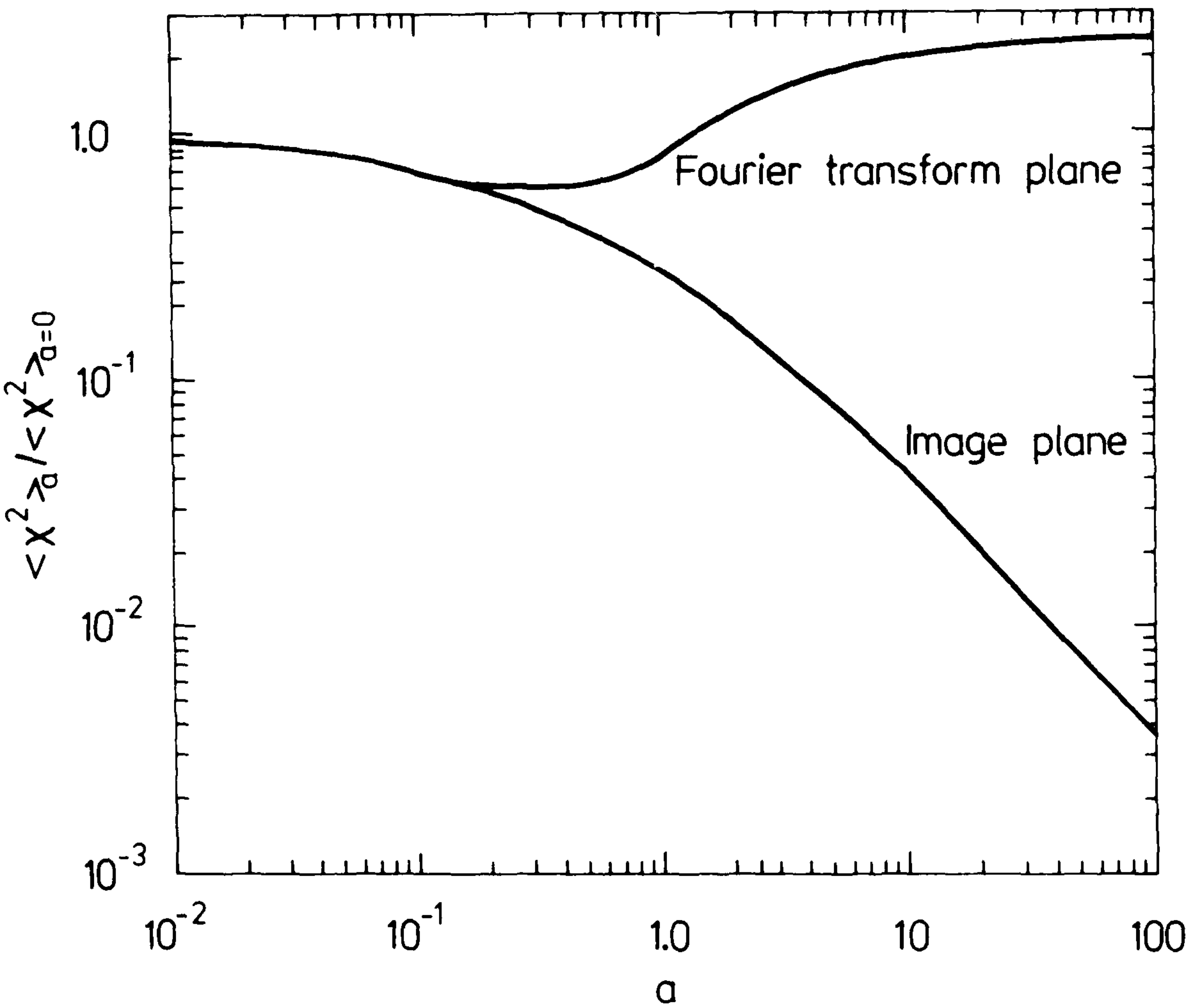

Fig. 3. Normalized Variance of Irradiance as a Function of "a" for Image Plane and Fourier Transform Plane Propagation Geometries. The quantity $a = k\sigma^2/2z$ for the image plane geometry and $k\sigma^2/2f$ for the Fourier transform plane geometry.

Examination of Eq. (5.17) reveals that in both the near field and far field of the transform lens, $\mu = \beta_i/\beta_r << 1$. In the far field, $\beta_r = z(1 - \frac{z}{f})$, whereas in the near field, $\beta_r = z$. As a result, the asymptotic limits for the Fourier plane on-axis log-amplitude variance are given by (for constant C_n^2)

$$\langle \chi^2 \rangle = 0.124\ k^{7/6}\ C_n^2 z_1^{11/6}\ , \quad z_1 >> k\sigma^2 \tag{5.18}$$

and

$$\langle \chi^2 \rangle = 0.307\ k^{7/6}\ C_n^2 z_1^{11/6}, \quad z_1 << k\sigma^2 \tag{5.19}$$

Equations (5.18) and (5.19) reveal that, in the far field (near field) of the transform lens, the on-axis variance of log-amplitude is given by the spherical wave (plane wave) variance for direct propagation over a distance z_1.[3] In contrast to the image-plane log-amplitude variance, no aperture averaging effects are obtained in the transform plane for near-field conditions. These asymptotic results can be explained as follows. In the far field, the wave impinging on the lens is nearly planar, hence, the resulting field on the right-hand side of the lens is approximately a spherical wave converging towards the focal point. Furthermore, in the far field, $\sigma << (z/k)^{1/2}$, and no aperture averaging is expected. Conversely, in the near field of the transform lens, the wave on the right-hand side of the lens is nearly collimated (i.e., a plane wave propagating parallel to the optic axis). As a result, the light that arrives on the optic axis in the transform plane results from a small region, of characteristic size, of the order $(f/k)^{1/2}$, about the axis of the lens. That is, the light arriving at a point on the optic axis in the transform plane is nearly planar and emanates from a region on the order of a single correlation patch in the plane of the lens. Thus, no aperture averaging effects are to be expected.

For arbitrary values of $a = k\sigma^2/2f$, Eq. (5.6) has been integrated numerically. The results are plotted in Fig. 3. These results indicate mild aperture averaging for $a \lesssim 1$.

6. ADAPTIVE OPTICS AND ABCD SYSTEMS

In this section, we present some results concerning the performance of adaptive-optics systems in an ABCD paraxial optical system between the input and output planes. For simplicity, we consider real ABCD systems, although the methods outlined here can be used directly to obtain the corresponding results for complex ABCD systems. To successfully implement adaptive-optics correction to either a laser transmitter system or to an imaging system, accurate measurements must be made of the phase errors associated with the appropriate propagation path.[7] Then, according to the principle of reciprocity, if the appropriate correction (i.e., the negative of the measured errors) is applied, the performance of the system will improve significantly. We consider conventional phase-conjugate adaptive-optics systems, where a beacon signal provides an estimate of the desired phase information. We assume that the beacon signal is a spherical wave that originates from some point in the output plane. Sometimes, the beacon location originates at the object of interest (i.e., the aim point of a laser transmitter or the object of the imaging system), and significant adaptive-optics improvement can be expected. Conversely, for some applications, the object of interest is moving relatively fast. Therefore, if light emitted from the object is used as a beacon, the adaptive-optics improvement is limited, because the beacon signal will propagate through a portion of the atmosphere different from that where adaptive-optics correction is desired.[7] These and other effects result in the various anisoplanatic degradation discussed in the literature.[7,8]

To be specific, we consider an adaptive-optics laser transmitter and seek to determine the resulting mean irradiance obtained at some point $\underline{p}_1$ in the output plane, where the beacon phase information originates from a point source located at some other point $\underline{p}_2$ in the output plane. For real ABCD systems, the mean irradiance at $\underline{p}_1$ is given by[2]

$$\langle I(\underline{p}_1)\rangle = \left|\frac{k}{2\pi B}\right|^2 \int d^2r \; \exp\left(-\frac{ik}{B}\,\underline{p}_1\cdot\underline{r}\right) \underline{K}(\underline{r})\; \Gamma_c(\underline{r}) \tag{6.1}$$

where

$$K(\underline{r}) = \int d^2R \; U_i(\underline{R} + \tfrac{1}{2}\underline{r})U_i^*(\underline{R} - \tfrac{1}{2}\underline{r}) \exp(- \frac{ikA}{B} \underline{r}\cdot\underline{R}) \tag{6.2}$$

$$\Gamma_c(\underline{r}) = \langle \exp[\psi_c(\underline{r}_1,\underline{p}_1;\underline{p}_2) + \psi_c^*(\underline{r}_2,\underline{p}_1;\underline{p}_2)]\rangle \tag{6.3}$$

U_i is the input wave function, A and B are the corresponding matrix elements for propagation through the entire ABCD system, and

$$\psi_c(\underline{r},\underline{p};\underline{p}_2) = \psi(\underline{r},\underline{p}) - iS_B(\underline{r},\underline{p}_2) \tag{6.4}$$

is the "corrected" turbulence-induced wave function. In Eq. (6.4)

$$\psi(\underline{r},\underline{p}) = \psi_1(\underline{r},\underline{p}) + \psi_2(\underline{r},\underline{p}) \tag{6.5}$$

is, through terms of second order in the fluctuations, the Rytov approximation to the field at $\underline{r}$ in the input plane resulting from a point source at $\underline{p}_1$ in the output plane, and $S_B(\underline{r},\underline{p}_2)$ is the phase of the beacon field at $\underline{r}$ resulting from a point source located at $\underline{p}_2$ in the output plane. The laser and beacon are assumed to have the same wavelength. Note that in phase-conjugation adaptive-optics imaging systems, part of the optical transfer function imposed on the system resulting from the random medium is also given by Γ_c.

For a given input wave function U_i and ABCD system, the deterministic quantity K, given by Eq. (6.2), can be obtained in a straightforward manner. In the remainder of this section, we concentrate on evaluating the statistical quantity Γ_c, the "corrected" mutual coherence function. If we assume that the beacon phase is a zero-mean random variable, application of Eq. (3.8) to Eq. (6.3) yields through terms of second order in the fluctuations that

$$\Gamma_c = \exp\,[-\langle|\psi_c|^2\rangle + F(\underline{r}_1,\underline{r}_2,\underline{p}_1;\underline{p}_2)] \tag{6.6}$$

where

$$F(\underline{r}_1,\underline{r}_2,\underline{p}_1;\underline{p}_2) = \langle \psi_{c1}(\underline{r}_1,\underline{p}_1;\underline{p}_2)\ \psi^*_{c1}(\underline{r}_2,\underline{p}_1;\underline{p}_2)\rangle \tag{6.7}$$

and

$$\psi_{c1}(\underline{r},\underline{p};\underline{p}_2) = \psi_1(\underline{r},\underline{p}) - iS_B(\underline{r},\underline{p}_2) \tag{6.8}$$

In obtaining Eq. (6.6)-(6.8), we have used the fact that for real ABCD systems

$$2\mathrm{Re}\langle\phi_2\rangle + |\psi_1|^2 = 0$$

which can be obtained directly from Eqs. (2.15) and (2.16) for $\underline{p} = \underline{\rho} = 0$. We have also used the fact that from statistical stationarity, $\langle\chi(\underline{r}_1,\underline{p}_1)S_B(\underline{r}_1,\underline{p}_2)\rangle = \langle\chi(\underline{r}_2,\underline{p}_1)S_B(\underline{r}_2,\underline{p}_2)\rangle$, and $\langle S(\underline{r}_1,\underline{p}_1)S_B(\underline{r}_1,\underline{p}_2)\rangle = \langle S(\underline{r}_2,\underline{p}_1)S_B(\underline{r}_2,\underline{p}_2)\rangle$.

Upon substituting Eqs. (6.7) and (6.8) into Eq. (6.6), neglecting the correlation between phase and log-amplitude, we can be show that

$$\Gamma_c = \exp\left[-\tfrac{1}{2}D_W(\underline{r}) - \tfrac{1}{2}D_{S_B}(\underline{r}) + d(\underline{r},\underline{p})\right] \tag{6.9}$$

where

$$D_W(\underline{r}) = \langle[\chi(\underline{r}_1,\underline{p}_1) - \chi(\underline{r}_2,\underline{p}_1)]^2\rangle + \langle[S(\underline{r}_1,\underline{p}_1) - S(\underline{r}_2,\underline{p}_1)]^2\rangle \tag{6.10}$$

is the wave structure function of a point source located at $\underline{p}_1$ in the output plane.[3]

$$D_{S_B}(\underline{r}) = \langle[S_B(\underline{r}_1,\underline{p}_2) - S_B(\underline{r}_2,\underline{p}_2)]^2\rangle \tag{6.11}$$

is the phase structure function of the beacon point source located at $\underline{p}_2$ in the output plane, and

$$d(\underline{r},\underline{p}) = \langle [S(\underline{r}_1,\underline{p}_1) - S(\underline{r}_2,\underline{p}_1)][S_B(\underline{r}_1,\underline{p}_2) - S_B(\underline{r}_2,\underline{p}_2)]\rangle$$

$$= \frac{1}{2}\,[D_{SS_B}(\underline{r},\underline{p}) + D_{SS_B}(-\underline{r},\underline{p}) - 2D_{SS_B}(\underline{p})] \tag{6.12}$$

where

$$D_{SS_B}(\underline{r},\underline{p}) = \langle [S(\underline{r}_1,\underline{p}_1) - S(\underline{r}_2,\underline{p}_2)]^2\rangle \tag{6.13}$$

is the structure function for point sources located at $\underline{p}_1$ and $\underline{p}_2$ in the output plane evaluated at the points $\underline{r}_1$ and $\underline{r}_2$ in the input plane [in Eq. (6.12), $D_{SS_B}(\underline{p}) = D_{SS_B}(0,p)$]. The quantity d is the correlation function between the residual phase difference of the object wave across the input plane and the beacon wave across the input plane. Because they are statistically stationary, D_W and D_{S_B} are independent of the location of the point source, in contrast to d, which is a function of $\underline{r}$ and $\underline{p}$. If the beacon is collocated with the object, and the measured beacon phase is a perfect estimate of the object phase, $\Gamma_c = \exp(-\frac{1}{2} D_A)$, where D_A is the log-amplitude structure function. This results in the best possible performance that can be obtained with conventional phase conjugation adaptive-optics.

The structure functions that appear in Eqs. (6.10)-(6.13) can be obtained directly from the results of Sections 2 and 4. By substituting Eqs. (2.16) and (2.17) into Eqs. (4.2) and (4.3), we can show that for isotropic power spectra

$$D_W(\underline{r}) = 8\pi^2k^2 \int_0^L dz \int_0^\infty dK\; K\Phi_n(K;z)\,[1 - J_0(K\alpha_1 r)] \tag{6.14}$$

$$D_{S_B}(r) = 4\pi^2k^2 \int_0^L dz \int_0^\infty dK\; K\Phi_n(K;z)\,[1 - J_0(K\alpha_1 r)]$$

$$\times\,[\,1 + \cos(\frac{K^2\beta}{k})] \tag{6.15}$$

and

$$D_{SS_B}(r) = 4\pi^2 k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)\left[\, 1 + \cos\left(\frac{K^2\beta}{k}\right)\right]$$

$$\times \left[1 - J_0(K|\alpha_1\underline{r} + \alpha_2\underline{\rho}|)\right] \tag{6.16}$$

where $\underline{r} = \underline{r}_1 - \underline{r}_2$, $\underline{\rho} = \underline{\rho}_1, \underline{\rho}_2$

$$\alpha_1(z) = \frac{B_{zL}(z)}{B} \tag{6.17a}$$

$$\alpha_2(z) = \frac{B_{0z}(z)}{B} \tag{6.17b}$$

and

$$\beta(z) = \frac{B_{0z}(z)\; B_{zL}(z)}{B} \tag{6.17c}$$

Equations (6.14)-(6.17) and Eq. (6.9) *represent the general solution for the* corrected MCF in a real ABCD system for use in calculating the adaptive-optics mean-irradiance profile when correlations between phase and log-amplitude are neglected (the usual case considered in the literature). If this latter restriction is relaxed, it can be shown that Γ_c contains an additional multiplicative factor of

$$\cos\left[B_{S_B\chi}(\underline{r}_1,\underline{r}_2;\underline{\rho}_1,\underline{\rho}_2) - B_{S_B\chi}(\underline{r}_2,\underline{r}_1;\underline{\rho}_1,\underline{\rho}_2)\right]$$

where $B_{S_B\chi}$, the correlation function of the beacon phase and object point log-amplitude, is given by

$$B_{S_B\chi}(\underline{r}_1,\underline{r}_2;\underline{\rho}_1,\underline{\rho}_2) = 2\pi^2 k^2 \int_0^L dz \int_0^\infty dK\, K\Phi_n(K;z)$$

$$\times \sin\left(\frac{K^2\beta}{k}\right) J_0(K|\alpha_1\underline{r} + \alpha_2\underline{\rho}|)$$

Within the framework of the present theory, the inclusion of the above term in Γ_c provides the coupling between phase-amplitude and log-amplitude in adaptive-optics systems. For certain applications (e.g., propagation conditions where the interaction between atmospheric turbulence and thermal blooming is important), the inclusion of this term is necessary to obtain a self-consistent analysis.

As an example, we consider the Kolmogorov spectrum in the inertial subrange[3] for propagation conditions where geometrical optics is valid. The cosine terms appearing on the right-hand side of Eqs. (6.15) and (6.16) can be replaced by unity. As a result, we obtain

$$\Gamma_c(r) = \exp[-S_c(r)] \tag{6.18}$$

where

$$S_c(\underline{r}) \simeq 2.91\, k^2 \int_0^L dz\, C_n^2(z) \Big[(\alpha_2 r)^{5/3} + (\alpha_1 p)^{5/3} - \frac{1}{2}\,|\alpha_2\underline{r} + \alpha_1\underline{p}|^{5/3} - \frac{1}{2}\,|\alpha_2\underline{r} - \alpha_1\underline{p}|^{5/3}\Big] \tag{6.19}$$

For line-of-sight propagation, $\alpha_1 = 1 - z/L$ and $\alpha_2 = z/L$, which, when substituted into Eq. (6.19), yields

$$S_c(r) = 2.19\, k^2 \int_0^L dx\, C_n^2(z)\Big\{[r(1 - z/L)]^{5/3} + (\theta z)^{5/3} - \frac{1}{2}\,|r(1 - z/L) + \theta z|^{5/3} - \frac{1}{2}\,|r(1 - z/L) - \theta z|^{5/3}\Big\} \tag{6.20}$$

where $\underline{\theta} = \underline{p}/L$ is the vector angular separation between the beacon and the object point. Equation 6.20 is identical to previous results found in the literature for common or angular anisoplanatism.[8]

7. CONCLUSIONS

Within the framework of the paraxial and the second-order Rytov approximation, we have derived explicit general expressions for calculating all second-order statistical moments of an optical wave propagating in a random media through an arbitrary complex paraxial ABCD system. These expressions are extensions of the corresponding results that apply both to line-of-sight propagation[3] and propagation through real ABCD systems.[2] The approach in this report (i.e., Huygens-Fresnel formulation of the field) uses complex ABCD matrices. This approach provides the most general form for analyzing propagation of any generalized paraxial wave through a complex paraxial optical system in the presence of random inhomogeneities distributed arbitrarily along the optical path. According to the Huygen's integral approach, an arbitrary optical wave propagated through an arbitrary complex paraxial system in a random medium, including all diffraction effects (even those resulting from the "Gaussian" limiting apertures), can be accomplished in one step, with knowledge only of the ABCD matrix elements of the system. Even though Huygen's integral has been used extensively in the literature, its general range of validity is perhaps not as well understood as it ought to be.

APPENDIX: DERIVATION OF F_1, F_2, AND F_3

In this appendix, we outline the derivation of the basic statistical quantities F_1, F_2, and F_3 introduced in Section 2. The method used is similar to that given in Ref. 9. The basic difference is that the zeroth-order paraxial Green function, rather than the free space function, is given by Eq. (2.8).

When the optical field in the absence of the inhomogeneities is point sources, the field U_o, as obtained from Eq. (2.4), is given by

$$U_o(\underline{r}_1,z) = \frac{C_1 e^{-ikz}}{B_{Oz}} \exp\left[-\frac{ik}{2B_{Oz}}\left(D_{Oz}r_1^2 - 2\underline{r}_1\cdot\underline{r} + A_{Oz}r^2\right)\right] \tag{A-1}$$

where $C_1 = -ik/2\pi$ and the point source is located at $(\underline{r},0)$.

A.1 DERIVATION OF F_1

If we substitute Eqs. (2.8) and (A-1) into Eq. (2.10), taking the statistical average and rearranging terms, we obtain

$$\langle U_2(\underline{\rho},L)\rangle = \frac{C_1 k^4 - ikL}{(4\pi)^2} \int d^3r_1 \int d^3r_2\, B_\varepsilon(\vec{r}_1,\vec{r}_2)(B_{Oz_2}B_{z_1L})^{-1}$$

$$\times H(\vec{r}_1,\vec{r}_2) \exp\left[-\frac{ik}{2B_{Oz_2}}\left(A_{Oz_2}r^2 - 2\underline{r}\cdot\underline{r}_2 + D_{Oz_2}r_2^2\right)\right]$$

$$\times \exp\left[-\frac{ik}{2B_{z_1L}}\left(A_{z_1L}r_1^2 - 2\underline{r}_1\cdot\underline{\rho} + D_{z_1L}\right)\right] \tag{A-2}$$

where

$$B_\varepsilon(\vec{r}_1 - \vec{r}_2) = \langle\varepsilon_1(\vec{r}_1)\varepsilon_1(\vec{r}_2)\rangle$$

is the correlation function of the fluctuations of the dielectric function [note that $B_n = \langle n(\vec{r}_1)n(\vec{r}_2)\rangle \approx 4B_\varepsilon$] and

$$H(\vec{r}_1,\vec{r}_2) = (B_{z_2z_1})^{-1} \exp[- \frac{ik}{2B_{z_2z_1}} (A_{z_2z_1} r_2^2 - 2\underline{r}_2\cdot\underline{r}_1 + D_{z_2z_1} r_1^2)] \qquad \text{(A-3)}$$

Next, we expand B_ε in terms of its two-dimensional Fourier transform[3]

$$B_\varepsilon\ (\vec{r}_1 - \vec{r}_2) = \int d^2K\ F_\varepsilon\ (\underline{K}, z_1 - z_2)\ \exp[-i\underline{K}\cdot(\underline{r}_1 - \underline{r}_2)] \qquad \text{(A-4)}$$

The quantity $F_\varepsilon(K,\Delta z)$ is non-zero only for $|\Delta z| \lesssim$ scale length of the inhomogeneities. We assume that the largest scale length is much smaller than any propagation distance of interest (i.e., the smallest distance between optical elements). Consider the integration over z_2. We only get a non-zero contribution to this integral for $|z_1 - z_2| \to 0$. Thus, $A_{z_1z_2} = D_{z_1z_2} \cong 1$, and $B_{z_1z_2} \to 0$. Hence, the function $H\ (\vec{r}_1,\vec{r}_2)$ is given by

$$H = \lim_{B_{z_2z_1} \to 0} [\exp(\frac{-ik(\underline{r}_2 - \underline{r}_1)^2}{2B_{z_1z_2}})]$$

$$= \frac{2\pi i}{k}\ \delta(\underline{r}_1 - \underline{r}_2) \qquad \text{(A-5)}$$

where $\delta(\underline{r})$ is the two-dimensional delta function. To obtain this limit, we assume that k has a small imaginary part, i.e., the wave propagates in a slightly absorbing medium. Substituting Eqs. (A-4) and (A-5) into Eq. (A-2) and noting that the z-dependence of the exponential factor is slowly varying with respect to that of F_ε, we obtain

$$\langle U_2(\underline{p},L)\rangle \cong \frac{C_1k^4(2\pi i/k)e^{-ikL}}{(4\pi)^2} \int d^2K \int_0^L dz_1 \int d^2r_1$$

$$\times\ (B_{0z_1}B_{z_1L})^{-1} \exp[- \frac{ik}{2B_{0z_1}} (A_{0z_1} r^2 - 2\underline{r}\cdot\underline{r}_1 + D_0 r_1^2)]$$

$$\times \exp[- \frac{ik}{2B_{z_1L}} (A_{z_1L} r_1^2 - 2\underline{r}_1\cdot\underline{p} + D_{z_1L} p^2)] \int_0^\infty dz\ F_\varepsilon(\underline{K},z) \qquad \text{(A-6)}$$

where $z_2 \cong z_1$ in the exponential terms on the left-hand side of Eq. (A-6), and, because $F_\varepsilon \cong 0$ for $z \gtrsim$ scale length of the fluctuations, the upper limit on the integral over z can be extended to infinity. From Ref. 3, we have

$$\int_0^\infty F_\varepsilon(\underline{K},z)dz = \pi\Phi_\varepsilon(K)$$

$$\simeq 4\pi\Phi_n(K) \tag{A-7}$$

where $\Phi_n(\underline{K})$ is the three-dimensional spectral density of the index of refraction fluctuations evaluated at $K_z = 0$.

Substituting Eq. (A-7) into Eq. (A-6), we obtain

$$\langle U_2(\underline{p},L)\rangle = C_1 k^4 (2\pi i/k) e^{ikL} \int_0^L dz \int d^2K \Phi_n(K) I \tag{A-8}$$

where

$$I = (B_{0z}B_{zL})^{-1} \exp\left[-\frac{ik}{2}\left(\frac{A_{0z}}{B_{0z}} r^2 + \frac{D_{zL}}{B_{zL}} p^2\right)\right] J \tag{A-9}$$

and

$$J = \int d^2r_1 \exp\left[-\frac{ik}{2}\left(\frac{A_{zL}}{B_{zL}} + \frac{D_{0z}}{B_{0z}}\right) r_1^2 - ik\underline{r}_1 \cdot \left(\frac{\underline{r}}{B_{0z}} + \frac{\underline{p}}{B_{zL}}\right)\right] \tag{A-10}$$

Because $M_{0L} = M_{0z}M_{zL}$, where $M_{z_1z_2}$ is the ABCD matrix for propagation from z_1 to z_2, we can show that $A_{zL}B_{0z} + B_{zL}D_{0z} = B_{0L}$ and, thus

$$J = \int d^2r_1 \exp\left[-\frac{ik}{2} \beta r_1^2 - ik\left(\frac{\underline{r}}{B_{0z}} + \frac{\underline{p}}{B_{zL}}\right) \cdot \underline{r}_1\right]$$

$$= (2\pi i/k)\left(\frac{B_{0z}B_{zL}}{B_{0L}}\right) \exp\left[\frac{ik\beta}{2}\left(\frac{\underline{r}}{B_{0z}} + \frac{\underline{p}}{B_{zL}}\right)^2\right] \tag{A-11}$$

where β is given by Eq. (2.18). To ensure convergence of this integral, we have again assumed that $\mathrm{Im}k < 0$. Combining Eqs. (A-10), (A-11), and (A-8) and using the relations

$$B_{zL} = A_{Oz}B_{OL} - B_{Oz}A_{OL}$$

and

$$B_{Oz} = B_{OL}D_{zL} - B_{zL}D_{OL}$$

we finally obtain

$$\langle U_2(\underline{p},L)\rangle = -U_o(\underline{p},L)\ \pi k^2 \int_0^L dz \int d^2K\Phi_n(K;z) \tag{A-12}$$

where

$$U_o(\underline{p},L) = \frac{C_1 e^{-ikL}}{B_{OL}} \exp\left[- \frac{ik}{2B_{OL}} (A_{OL}r^2 - 2\underline{r}\cdot\underline{p} + D_{OL}p^2)\right] \tag{A-13}$$

is the field of the point source in the absence of the inhomogeneities, and we have tacitly assumed a slowly varying dependence of Φ_n on propagation distance. Thus, from Eqs. (A-12), (A-13), (2.12), and (2.11a), we obtain the results given by Eq. (2.15).

A.2 DERIVATION OF F_2 AND F_3

The derivation of both F_2 and F_3 follows exactly the same steps that result in Eq. (19) of Ref. 9, except that Eq. (2.8) is used as the zeroth-order Green function instead of the free space Green function. The final result of such a procedure leads directly to Eqs. (2.16) and (2.17).

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